Numerical validation of casting flow analysis using the Cut Cell method: A grid dependency study

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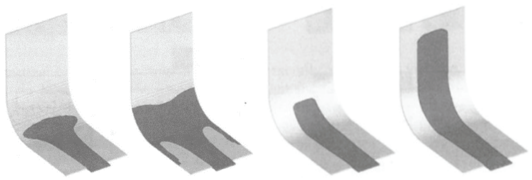
Abstract: Accurate simulation of the mold-filling process is essential for producing high-quality castings, yet generating an appropriate grid for complex geometries remains a significant challenge. Conventional Cartesian grids, while simple to generate, introduce stair-stepped boundaries on curved surfaces. This geometric distortion leads to artificial momentum loss and alters the molten metal's flow pattern. This study presents a numerical validation of the Cut Cell method, which is designed to overcome these limitations. A grid dependency study was conducted, comparing the casting flow analysis results from a standard Cartesian grid, a refined Cartesian grid, and a grid implementing the Cut Cell method. The findings reveal that the Cut Cell method provides highly accurate results even with a coarse mesh. Notably, it demonstrates significantly less momentum loss compared to even the finely refined Cartesian grid. This concludes that simply increasing grid density is insufficient to compensate for inherent momentum loss, validating the Cut Cell method as a more efficient and accurate approach for high-fidelity casting simulations.

Keywords: casting flow analysis; Cut Cell method; cartesian grid; momentum loss; grid dependency

1 Introduction

Casting is a manufacturing process used to shape molten metal into desired forms. This process encompasses two main phases: mold filling and solidification. Casting flow analysis plays a crucial role in predicting defects by evaluating the adequacy of the gating system design. Ensuring accurate casting flow simulations is important.

Casting flow is a two-phase flow with a free surface of air and molten metal. Traditional numerical methods, such as MAC (Marker And Cell), SMAC (Simplified Marker And Cell), or SOLA-VOF (A SOLution Algorithm for transient fluid flow - Volume Of Fluid) have been used in the field of casting flow [1-3]. These methods use the Cartesian grid system. The Cartesian grid system offers several advantages, including the ease of mesh generation without requiring specialized techniques and relatively quick mesh generation [4, 5]. However, it has limitations. When a cell contains the interface between a casting and a mold, it is treated by volume fraction as only one material of two, resulting in stepped grid domains within the Cartesian grid system. Consequently, original geometries with sloped or curved surfaces cannot be accurately represented in this grid system [6-8]. To address this limitation of the Cartesian grid, an alternative approach is to utilize a body-fitted grid [9-11]. Mampaey et al. conducted experimental and numerical analyses of castings using gating systems with curved, straight, and step-shaped configu- rations, comparing them separately [12]. They demonstrated differences in the flow patterns between gates with curves or slopes and those with a step-shaped configuration. To address this, Mampaey employed a structured non-orthogonal mesh in orthogonal curvilinear coordinates. Hetu et al. applied the Finite Element Method (FEM) to analyze casting flow and solidification [13]. Hong et al. developed the SIMPL-BFC-VOF method, showcasing its capability to accurately simulate mold filling of castings with complex shapes featuring curved surfaces and thin walls in a Body-Fitted Coordinate (BFC) system [14]. Fig. 1 presents the results of flow analysis in an L-curved cavity with molten metal injected at 1 m/s using both a Cartesian grid system and a BFC system [Figs. 1(a, b)] [15]. Without any special treatment in the Cartesian grid system, the flow results are inaccurate due to the stair-step grid which does not match the original geometric structure. However, the BFC method requires skilled techniques to generate the correct grid and can be time-consuming during the grid generation phase, occasionally exceeding the calculation time required for flow analysis [16, 17]. Furthermore, since most castings have complex geometric structures with slopes and curves, applying the BFC system in casting flow simulations is challenging. Cartesian coordinates, on the other hand, have been used predominantly in casting process simulations due to their relatively easy and fast grid generation [18].



(a) Cartesian grid system (b) BFC system

Fig. 1: Filling sequence of L-curved cavity with 1m/s [15].

Increasing the number of grids is the easiest and simple method to reduce shape distortion in the Cartesian grid system. Many grids create a calculation domain closer to the original shape. Another strategy involves trimming surface cells [19]. Recent advancements in the cell trimming method have focuse d on enhancing its stability, accuracy, and applicability to complex flow regimes. To address the numerical instabilities arising from the 'small cell problem' inherent in cell trimming, Taylor et al. have proposed a high-order accurate discontinuous Galerkin method incor- porating a state redistribution technique to ensure stability and accuracy on cut-cell grids [20]. For the accurate resolution of viscous flows, Kleb et al. developed a sophisticated solver that integrates the cut-cell framework with adaptive mesh refinement (AMR) to precisely capture boundary layer physics [21]. Furthermore, new approaches have emerged to tackle high-Reynolds-number turbulent flows; Abe et al., for instance, enhanced predictive efficiency by incorporating a wall-stress model into the Cartesian cut-cell method [22], while Kleb et al. improved numerical stability and convergence by utilizing a Jacobian-Free Newton-Krylov (JFNK) method [23]. Lin applied the Cartesian cut cell approach to simulate moving bodies in a free-surface flow in an orthogonal grid system [24]. Choi et al. applied the PCT-based Cut Cell method to casting flow analysis [25].

This study validates the effectiveness of the Cut Cell method by quantitatively comparing its casting flow analysis results against those from a conventional Cartesian grid and a refined Cartesian grid.

2 Numerical method

The casting flow is modeled as unsteady, in-compressible, and viscous two -phase flow with a free surface. The dynamic viscosity of the molten metal varies significantly with temperature below the liquidus point but remains relatively stable above it. As this study focuses on the mold-filling stage where the temperature is assumed to be above the liquidus point, the dynamic viscosity is treated as a temperature-independent constant.

The fluid flow is governed by the three-dimensional, unsteady, incompressible Navier-Stokes equations. The SOLA method is employed for the flow field calculation, where the pressure and velocity fields are updated using the PSOR (Point Successive Over-Relaxation) method. To track the free surface inherent in this two-phase flow, the VOF method is utilized. The advection of the VOF function is handled by the Donor and Acceptor Flux Approximation (DAFA) for interface reconstruction.

In a Cartesian grid system for casting analysis, each cell must represent either the mold or the casting. This approach, however, struggles to accurately resolve the complex geometry where the mold boundary intersects a cell. To address this, the Cut Cell method is employed. This method introduces a geometric openness function, θ, to represent the fraction of each cell occupied by the casting. A cell is considered part of the solid mold when $\theta = 0$ and is the full casting when $\theta = 1$. Cells with $0 < \theta < 1$ are defined as cut cells. θ_c shows the volume fraction of the casting at the center of the cell, and that value is used in the continuity equation and VOF equation. The area fractions on each face ($\theta_{R,L,F,K,T,B}$ for the x, y, and z axes, respectively) represent the open area available for flow and are incorporated into the momentum equations to accurately model fluxes across partially obstructed faces. These values indicate the extent to which each cell face is occupied by the casting geometry. Based on the values of θ, mass conservation is calculated for the portion of the cell that is filled with the casting.

The governing equations are formulated for the Cut Cell method by incorporating the θ function into the standard conservation equations.

The governing equations are as follows:

Continuity equation

$$\nabla \cdot (\theta \vec{u}) = 0 \tag{1}$$

Momentum equation

$$\frac{\partial(\theta \vec{u})}{\partial t} + \nabla \cdot (\theta \vec{u}\vec{u}) = -\frac{\theta}{\rho}\nabla p + \nu\theta\nabla^2\vec{u} + \theta\vec{g} \tag{2}$$

Volume of Fluid equation

$$\frac{\partial(\theta f)}{\partial t} + \nabla \cdot (\theta f \vec{u}) = 0 \quad (3)$$

Where $\vec{u}$ is the velocity vector and p is pressure, ρ is the density of molten metal, $\vec{g}$ is the acceleration of gravity, ν is the dynamic viscosity, f is the volume fraction of fluid in a cell, and t is time.

3 Simulation results and analysis

The most straightforward approach to address issues related to shape distortion in a Cartesian grid system is to increase the grid resolution. Recent advances in computer performance have made it relatively easy to generate over ten million meshes in the Cartesian grid system. For castings with numerous sloped and curved surfaces, creating a large number of meshes can result in a computational domain that closely approximates the original shape.

In this study, the existing numerical method for casting flow analysis was conducted using each a coarse Cartesian grid and a fine Cartesian grid. Furthermore, the results with the application of the Cut Cell method with a coarse Cartesian grid were compared. How increasing the grid resolution affects energy losses due to stair-step grids was investigated and the differences compared to the results obtained using the Cut Cell method was examined.

3.1 Die-casting with a sloped ingate of 45°

A fluid analysis was performed on a die-casting product featuring a 45-degree inclined ingate [Fig. 2]. The computational domain was discretized using two Cartesian grids of different resolutions, as illustrated in Fig. 3. The fine grid [Fig. (3b)] was generated by refining the coarse grid [Fig. (3a)] by a factor of 2.5 in each spatial dimension, leading to an approximately 16-fold increase in the total cell count. In the coarse grid, the inclined ingate is represented by a distinct staircase shape. Although the fine grid provides a closer approximation to the original geometry, a detailed view reveals that the ingate is still composed of numerous small steps.

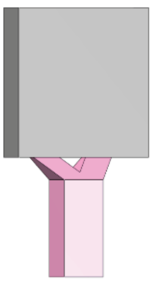


Fig. 2: 3D model.

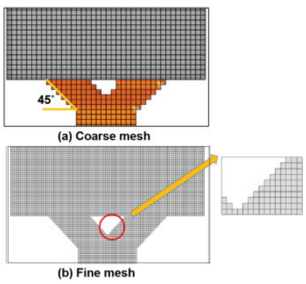


Fig. 3: Cartesian Grids.

The molten metal is injected at a speed of 1000 cm/s. Due to the smaller cross-sectional area of the ingate, the flow significantly accelerates as it passes through this section. The rectangular sprue is aligned with the coordinate axes, allowing for a perfect represen-tation in the Cartesian mesh. The detailed analysis conditions and grid information for each case are summarized in Table 1.

Table 1. Analysis conditions.

Cast	A356
Mold	SKD61
Pouring rate	1,000cm/s
Coarse mesh	48×16×76
Fine mesh	120 × 40 ×190

Fig. 4-6 compare the casting flow analysis at mold filling rates of 24%, 30%, and 40% for three different simulation cases. Fig. 4 shows the results using a conventional method on a coarse grid (58,368 cells), where the inclined ingate is represented by a distinct staircase pattern.

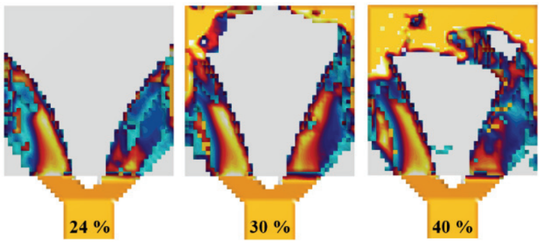


Fig. 4: Results of the conventional method with a coarse grid

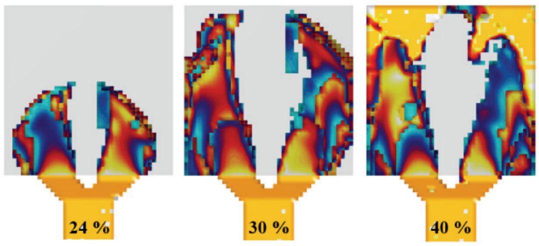


Fig. 5: Results of the conventional method with a fine grid

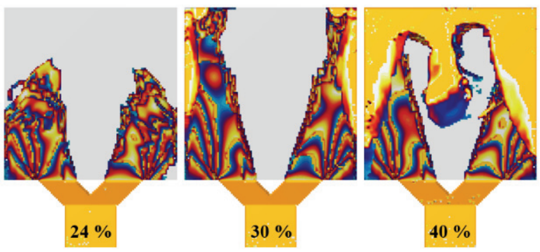


Fig. 6: Results of the Cut Cell method with a coarse grid

Fig. 5 depicts the results from the same method but on a fine grid, refined by a factor of 2.5 in each dimension for a total of 912,000 cells. Finally, Figure 6 illustrates the results obtained using the Cut Cell method on the same coarse grid as the first case.

The simulation results highlight significant differences in the flow patterns produced by each method [Figs. 4-6]. Fig. 4, representing the conventional method on a coarse grid, shows a non-physical flow path. The molten metal exits the ingate almost vertically instead of at the intended 45-degree angle, indicating severe momentum loss from the coarse staircase geometry. This prevents the flow from properly reaching the upper corners of the cavity. While grid refinement [Fig. 5] improves the injection angle and enables better filling, the underlying geometry is still approximated by numerous small steps. The most accurate representation of the intended flow path, however, is achieved with the Cut Cell method [Fig. 6]. Even on the coarse grid, the molten metal is correctly injected at a 45-degree angle. It maintains its velocity without the artificial deceleration caused by the stepped geometry and flows directly toward the upper regions of the cavity. This comparison demonstrates a key limitation of applying a conventional method to a Cartesian grid: the staircase approximation of inclined surfaces acts as a series of artificial obstacles, causing non-physical momentum dissipation. Although refining the grid reduces the size of these obstacles and improves the result [Fig. 5], it does not eliminate the fundamental issue and significantly increases computational cost. In contrast, the Cut Cell method [Fig. 6] is shown to capture the correct flow physics—preserving the injection angle and momentum—even on a coarse grid. This indicates that the Cut Cell method can achieve the accuracy of a fine-grid simulation without the associated computational expense.

3.2 Gravity casting with a curved surface

This study analyzes a gravity casting product characterized by a complex curved surface, with the 3D model shown in Fig. 7. For the simulation, the computational domain was discretized using two Cartesian grids of differing resolutions, as illustrated in Fig. 8. The coarse mesh [Fig. 8(a)] consists of 105,600 cells (60×80×22), while the fine mesh [Fig. 8(b)] was generated by refining the coarse mesh by a factor of two in each dimension, resulting in a total of 844,800 cells (120×160×44). A key challenge with this meshing approach is the representation of the product's curved surface. The coarse grid clearly approximates the curvature with a distinct staircase pattern [Fig. 8(a)]. Although the refined grid provides a closer fit to the actual geometry [Fig. 8(b)], a detailed inspection reveals that the surface is still fundamentally represented by a series of smaller, discrete steps. The specific conditions are summarized in Table 2.

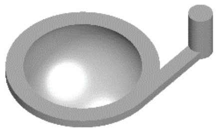


Fig. 7: 3D model

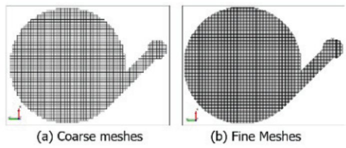


Fig. 8: Cartesian Grids

Table 2. Analysis conditions

Cast	A356
Mold	FURAN
Pouring rate	100cm/s
Coarse mesh	60× 80 × 22
Fine mesh	120 × 160 ×44

Three simulation cases are compared to evaluate the numerical method's performance: (1) a conventional method on a coarse grid [Fig. 9], (2) the Cut-Cell method on the same coarse grid [Fig. 10], and (3) the conventional method on a fine grid with eight times the total cell count [Fig. 11]. Significant differences in the resulting flow patterns are observed. In the conventional coarse-grid simulation [Fig. 9], the flow fails to adhere to the curved surface due to the staircase approximation. The molten metal separates from the wall and flows incorrectly toward the upper surface of the hemisphere. In contrast, the Cut Cell method on the same coarse grid [Fig. 10] correctly captures the flow physics, with the molten metal following the circular path of the cavity. This difference is particularly evident at the 50% filling stage: in the Cut Cell result, the flow has completed a full loop and rejoined at the ingate, whereas in the conventional case, it has not yet reached this point. While the fine-grid simulation [Fig. 11] shows an improvement over the coarse grid, its forward progression remains slower than that of the Cut Cell result.

The results demonstrate that the staircase representation in the conventional Cartesian method introduces non-physical artifacts, primarily artificial momentum dissipation. This causes the flow to deviate from the intended path and can lead to inaccurate predictions of filling behavior and gas porosity locations. While uniformly refining the grid can mitigate these errors, it does not entirely resolve the issue and comes at a significant computational expense. The Cut Cell method, however, addresses the geometric inaccuracy directly. It accurately predicts the filling behavior even on a coarse grid, proving to be a more efficient and physically accurate approach for this type of problem.

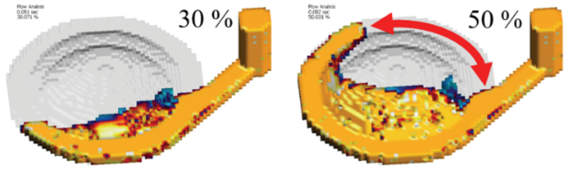


Fig. 9: Results of the conventional method with a coarse grid

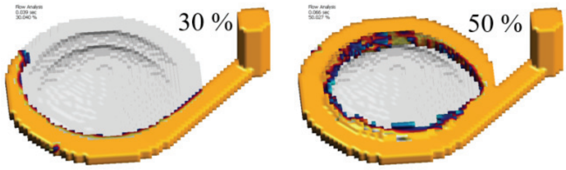


Fig. 10: Results of the Cut Cell method with a coarse grid

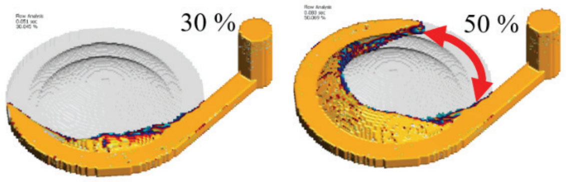
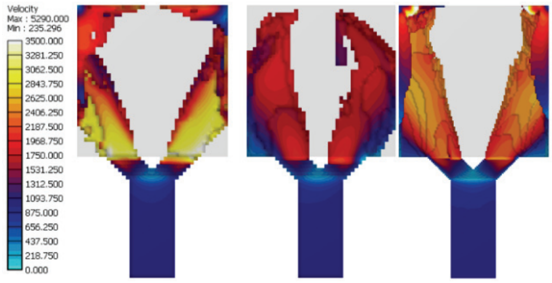


Fig. 11: Results of the conventional method with a fine grid

3.3 Momentum loss

Fig. 12 presents a comparison of velocity contours for the die-casting product analyzed in Section 3.1, at a 30% filling rate, to analyze the momentum loss in each numerical case. The figure compares the results from the Cut Cell method on a coarse grid [Fig. 12 (a)], the conventional method on a coarse grid [Fig. 12 (b)], and the conventional method on a fine grid [Fig. 12 (c)]. In this simulation, the molten metal is injected at 1000 cm/s and is expected to accelerate as it passes through the ingate, which has a cross-sectional area approximately one-third that of the injection area. Based on the principle of continuity, the velocity should ideally increase to around 3000 cm/s. A quantitative analysis of the velocity just past the ingate reveals significant discrepancies between the methods, which directly indicates the degree of artificial momentum loss. The Cut Cell simulation [Fig. 12 (a)] shows a peak velocity of approximately 3100 cm/s, aligning well with the theoretical expectation. In contrast, the conventional method on the same coarse grid [Fig. 12 (b)] exhibits a severely reduced velocity of only 1600 cm/s, indicating a momentum loss of nearly 50%. While grid refinement improves the result [Fig. 12 (c)], the velocity only recovers to approximately 2400 cm/s.

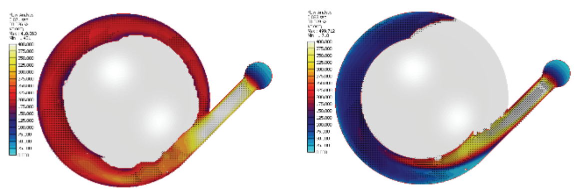


(a) (b) (c)

Fig. 12: Velocity color contour for each method at a 30% mold filling rate. (a) Cut cell method in a coarse grid system, (b) Conventional method in a coarse grid system, and (c) Conventional method in a fine grid system.

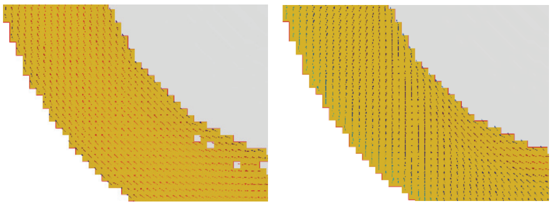
This comparison clearly demonstrates that the staircase approximation in the conventional method introduces significant non-physical momentum dissipation. Although refining the grid can mitigate this issue, it still fails to fully preserve the flow's momentum and comes at a high computational cost. The Cut Cell method, however, minimizes this artificial momentum loss even on a coarse grid by accurately representing the geometry. This proves that the Cut Cell method is more effective at preserving momentum than simple grid refinement, leading to a more physically accurate simulation.

To further investigate the performance of each method, a fine-grid comparison was conducted, with the results at a 50% filling rate shown in Fig. 13 and 14 (from Section 3.2). The velocity contours in Figure 13 reveal that the average velocity in the Cut Cell simulation (a) is approximately 225 cm/s, significantly higher than the 125 cm/s observed in the conventional method (b). Figure 14 provides a detailed view of the velocity vectors, explaining the cause of this discrepancy. In the Cut Cell simulation, the velocity vectors correctly follow the intended circular path of the geometry, even on the underlying staircase grid. Conversely, the vectors in the conventional method are forced to align perpendicularly with the stepped grid faces. This misdirection of the local flow is the primary source of the artificial momentum loss and directly explains the non-physical flow separation seen in the overall results (e.g., Figs. 9 and 11).



(a) Cut cell method (b) Conventional method

Fig. 13: Velocity color contour for each method in a fine grid system at a 50% mold filling rate.



(a) Cut cell method (b) Conventional method

Fig. 14: Velocity vector for each method in a fine grid system at a 50% mold filling rate.

In conclusion, this study demonstrates that while grid refinement can mitigate the errors caused by the staircase approximation in conventional methods, it cannot eliminate the fundamental source of the non-physical behavior. Even on a fine grid, the conventional method still forces the flow to navigate a series of discrete steps, leading to inherent momentum dissipation. The Cut Cell method, in contrast, geometrically modifies the governing equations to account for the true boundary shape within each cell. This allows it to preserve the physical flow path and momentum far more effectively, yielding accurate results without the substantial computational expense of extreme grid refinement. This establishes the Cut-Cell method as a more robust and efficient approach for simulating flows with complex, curved geometries.

4 Conclusions

This study conducted a grid dependency analysis to validate the Cut Cell method as an effective approach for simulating complex casting flows. The investigation confirmed that conventional methods on Cartesian grids introduce non-physical momentum loss due to the staircase approximation of curved and inclined geometries. It was demonstrated that while refining the grid can mitigate this issue, it fails to eliminate the fundamental source of the error and comes at a significant computational cost. In contrast, the Cut Cell method consistently produced physically accurate flow patterns, preserving momentum even on a coarse grid.

The central conclusion of this work is that simply increasing the number of cells is an insufficient and inefficient solution for the geometric inaccuracies inherent in the conventional Cartesian grid method. The Cut Cell method addresses the problem at its source by geometrically modifying the governing equations within cells intersected by the boundary. Therefore, this study concludes that the Cut Cell method is a robust, efficient, and more physically accurate alternative, enabling high-fidelity casting simulations without the prohibitive computational expense of extreme grid refinement.

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Conflicts of interest:

The authors declare that they have no known competing financial interests.

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